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Certain Results of Pre Open Sets Via Soft Bitopological Spaces

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ABSTRACT

In this article the first goal is to offer the definition a neoteric generalized to soft open set in soft bitopological spaces shortly (S. B.T. S) term soft (1,2)*-pre-c-open set is done through integration soft $\tilde{\tau}_1\tilde{\tau}_2$-closed and soft.(1,2)*-pre-open. set. The relationship between this kind of soft set with other kinds of soft sets also we explained by drawing a diagram. And we proved sundry facts, results and theory. We savvy of soft (1,2)*-pre-c-closure (S.(1,2)*-pre-c-cl), soft (1,2)*-pre-c-interior (s. (1,2)*-pre-c-int) and soft (1,2)*-pre-c-neighborhood S.(1,2)*-pre-c-nbh) are verified and its properties weredescribed. We also found many features and characterizatics of this kind of soft sets. In this article, we want number of definitions, theorems and facts, the superior are: Molodtsov is introducing a notion of the soft set. And solve numerous issue of it and he implementation the concept of soft set in practical trends and numerous applied from them theory of Theory of Probability, Smoothness of functions, Rieman integration Operations research and Game theory, etc., Shabir and Naz study the soft topological spaces (S. T. S). Senel and Cagman are offer the nation of soft bitopological spaces.

1. Introduction

Molodtsov [1] in 1999 he is the first to introduce the notion of soft set. He manage applied the s. set theory in to important numerous trends. In 2011 ecstatic and present the view of S. T. S. by Naz and Shabir [3] and other [2],[5] and [12]. In 2014 ecstatic and studied S. B.T. S. by Çagman and Senel [4] with a fixed set of parameters over an initial universe set. In 2025 [8] Asaad Adil Abdulhadi, Marwa Makki Dahham and Haider Jebur Ali we learned s. (1,2)*-pre-clo. sets. and s. (1,2)*-pre-o. sets in S. B.T. S.. respectively. Mahmood, and Abdul-Hadi [9] define soft (1,2)*-locally indiscrete and the condition of property (I) in S. B.T. S.. In such treatise we describe and research new sort of soft open sets in S. B.T. S. known as s. (1,2)*-pre-c-o. sets and s. (1,2)*-pre-c-neighborhood. further some basic properties of this type of soft open sets have also been examined.

1. Mathematic Preliminaries

during this treatise H is initial universe set, P(H) is the power set of H, Γ is parameters set. And J⊆Γ.

Definition 2.1[13]: pair (K,J) of s. set over H , parameters J is a non-idle subset of Γ, and represent K is a mapping specified by K:J⟶P(H).

Definition 2.2 [6]: A soft point of a soft set (K,J) in H. if m∈J then K(m)={n} for several $n \in H$ and $K(ḿ) = \emptyset$, $\forall\, ḿ \in J\backslash\{n\}$ and be represented by $\tilde{n} = (m, \{n\})$.

Definition 2.3[3]: A soft topology shortly (S. T.) on H and a collection $\tilde{\tau}$ is a s. subsets of $\widetilde{H}$ possessing the qualities listed below:

(i) $\widetilde{\emptyset} \tilde{\in} \tilde{\tau}$ and $\widetilde{H} \tilde{\in} \tilde{\tau}$.
(ii) If $(L_1, \Gamma), (L_2, \Gamma) \tilde{\in} \tilde{\tau}$, then $(L_1, \Gamma) \tilde{\cap} (L_2, \Gamma) \tilde{\in} \tilde{\tau}$.
(iii) If $(L_j, \Gamma) \tilde{\in} \tilde{\tau}, \forall\, j \in \Omega$, $\bigcup_{j\in\Omega}(L_j, \Gamma) \tilde{\in} \tilde{\tau}$.

The trine acting $(H, \tilde{\tau}, \Gamma)$ is named a soft topological spaces (S. T. S.) on H. where $\tilde{\tau}$ are named soft open (s. o.) sets on H. The soft closed (s. clo.) sets is acomplement of a s. o. set.

Definition 2.4[8]: let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be two diverse S. T. on H, and H be a non- idle set. Then $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ is named asoft bitopological spaces (S. B. T. S.) on H.

Definition 2.5[4]: In S. B. T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ A s. subset (V, Γ) of is named soft $\tilde{\tau}_1\tilde{\tau}_2$-open (s. $\tilde{\tau}_1\tilde{\tau}_2$-o.) if $(V, \Gamma) = (V_1, \Gamma) \tilde{\cup} (V_2, \Gamma)$ for $(V_1, \Gamma) \tilde{\in} \tilde{\tau}_1$, $(V_2, \Gamma) \tilde{\in} \tilde{\tau}_2$. When soft $\tilde{\tau}_1\tilde{\tau}_2$-open (s. $\tilde{\tau}_1\tilde{\tau}_2$-clo.) set is acomplement of a s. $\tilde{\tau}_1\tilde{\tau}_2$-clo (s. $\tilde{\tau}_1\tilde{\tau}_2$-o.) set in $\widetilde{H}$.

The all group of s. $\tilde{\tau}_1\tilde{\tau}_2$-o. sets in $\widetilde{H}$ not necessarily a S. T. on H it can be viewed by next example:

Example 2.6: Let $H = \{f, p, d, g, n\}$ and $\Gamma = \{r_1, r_2\}$, and let $\tilde{\tau}_1 = \{\widetilde{H}, \widetilde{\emptyset}, (V_1, \Gamma)\}$ and $\tilde{\tau}_2 = \{\widetilde{H}, \widetilde{\emptyset}, (V_2, \Gamma)\}$ be S.T. over H, where $(V_1, \Gamma) = \{(r_1, \{f, p, d\}), (r_2, \{\widetilde{H}\})$. $(V_2, \Gamma) = \{(r_1, \{d, g, n\}), (r_2, \{\widetilde{H}\})\}$. The s. sets in $\{\widetilde{H}, \widetilde{\emptyset}, (V_1, \Gamma), (V_2, \Gamma)\}$ are s. $\tilde{\tau}_1\tilde{\tau}_2$-o. set in $\widetilde{H}$. Since $(V, \Gamma) = (V_1, \Gamma) \tilde{\cap} (V_2, \Gamma) = \{(r_1, \{d\}), (r_2, \{\widetilde{H}\})\}$, but (V, Γ) is not s. $\tilde{\tau}_1\tilde{\tau}_2$-o. set in $\widetilde{H}$. Thus $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ is not S. topology on H.

Definition 2.7[4]: If $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ be a S. B.T. S., and $(V, \Gamma) \tilde{\subseteq} \widetilde{H}$. Then:

(i) $\tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma) = \tilde{\cup}\{(K, \Gamma): (K, \Gamma)$ is s. $\tilde{\tau}_1\tilde{\tau}_2$-o. set in $\widetilde{H}$ and $(K, \Gamma) \tilde{\subseteq} (V, \Gamma)\}$ is named the soft $\tilde{\tau}_1\tilde{\tau}_2$-interior of (V, Γ).
(ii) $\tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma) = \tilde{\cap}\{(F, \Gamma): (F, \Gamma)$ is s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set in $\widetilde{H}$ and $(V, \Gamma) \tilde{\subseteq} (F, \Gamma)\}$ is named the soft $\tilde{\tau}_1\tilde{\tau}_2$-closure of (V, Γ).

Theorem 2.8[9]: If $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ be S. B. T. S., and $(V, \Gamma), (D, \Gamma) \tilde{\subseteq} \widetilde{H}$ Then:

(i) $\tilde{\tau}_1\tilde{\tau}_2 int(D, \Gamma) \tilde{\subseteq} (D, \Gamma)$, $(D, \Gamma) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 cl(D, \Gamma)$.
(ii) If $\{(V_\varepsilon, \Gamma)\}_{\varepsilon\in\Omega}$is a combination of s. $\tilde{\tau}_1\tilde{\tau}_2$-o. sets in $\widetilde{H}$, then also $\bigcup_{\varepsilon\in\Omega}(V_\varepsilon, \Gamma)$.
(iii) If $\{(V_\varepsilon, \Gamma)\}_{\varepsilon\in\Omega}$ is a combination of s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. sets in $\widetilde{H}$, then also $\bigcap_{\varepsilon\in\Omega}(V_\varepsilon, \Gamma)$.
(iv) $\tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma)$ is s. $\tilde{\tau}_1\tilde{\tau}_2$-o. set in $\widetilde{H}$ and $\tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma)$ is s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set in $\widetilde{H}$.
(v) (V, Γ) is s. $\tilde{\tau}_1\tilde{\tau}_2$-o. iff $\tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma) = (V, \Gamma)$.
(vi) $(V, \Gamma)$is s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. iff $\tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma) = (V, \Gamma)$.
(vii) $\tilde{\tau}_1\tilde{\tau}_2 int\ (\tilde{\tau}_1\tilde{\tau}_2 int\ (V, \Gamma)) = \tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma)$ and $\tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma)) = \tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma)$.
(viii) $\widetilde{U} - (\tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma)) = \tilde{\tau}_1\tilde{\tau}_2 cl(\widetilde{U} - (V, \Gamma))$ and $\widetilde{U} - (\tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma)) = \tilde{\tau}_1\tilde{\tau}_2 int(\widetilde{U} - (V, \Gamma))$.
(ix) If $(V, \Gamma) \tilde{\subseteq} (D, \Gamma)$, then $\tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 int(D, \Gamma)$.
(x) If$(V, \Gamma) \tilde{\subseteq} (D, \Gamma)$,then $\tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 cl(D, \Gamma)$.

Definitions 2.9[7][9][11]: In S. B.T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ and (V, Γ) be s. subset of H Then:

(i) (V, Γ) is term s.(1,2)*-α-o. if $(V, \Gamma) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma)))$.
(ii) (V, Γ) is term s.(1,2)*-pre-o. if $(V, \Gamma) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma))$.
(iii) (V, Γ) is term s.(1,2)*-pre-clo. if $(V, \Gamma) \tilde{\supseteq} \tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma))$.
(iv) (V, Γ) is term s.(1,2)*-semi-o. if $(V, \Gamma) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 cl\ (\tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma))$.
(v) (V, Γ) is term s. (1,2)*-b-o. if $(V, \Gamma) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2\ int(\tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma)) \tilde{\cap} \tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(V, \Gamma))$.
(vi) (V, Γ) is term s. (1,2)*-β-o. if $(V, \Gamma) \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(V, \Gamma)))$.

Definition 2.10[8]: A S. B. T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ is termed a soft (1,2)*-$\widetilde{T}_1$-spaces shortly (S.(1,2)*-$\widetilde{T}_1$-S.). if for any two variant soft points $\widetilde{m}$ and $\tilde{r}$ in $\widetilde{H}$, there are two s. $\tilde{\tau}_1\tilde{\tau}_2$-o. set (V_1, Γ) and (V_2, Γ) in $\widetilde{H}$ such that $\widetilde{m} \tilde{\in} (V_1, \Gamma)$, $\tilde{r} \tilde{\notin} (V_1, \Gamma)$ and $\tilde{r} \tilde{\in} (V_2, \Gamma)$, $\widetilde{m} \tilde{\notin} (V_2, \Gamma)$.

Theorem 2.11[8]: Let $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ be a S. B. T. S.. Then every soft point is a s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. if and only if $(H, \tilde{\tau}_{1.}, \tilde{\tau}_{2.}, \Gamma)$ is a s. (1,2)*-$\widetilde{T}_1$-spaces.

Definition 2.12[9]: $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ be S. B.T. S. is termed a soft (1,2)*-locally indiscrete, if every s. $\tilde{\tau}_1\tilde{\tau}_2$-o. set over H is a s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set over H.

Definition 2.13[8]: A S. B.T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ is said to have property (I) if the intersection of the finite number of s. $\tilde{\tau}_1\tilde{\tau}_2$-o. set is s. $\tilde{\tau}_1\tilde{\tau}_2$-o.

Proposition 2.14[8]: A S. B.T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ with property (I), then the collection of all s. $\tilde{\tau}_1\tilde{\tau}_2$-o. sets in $\widetilde{H}$ froms a S. T. on $\widetilde{H}$.

3. Soft (1,2)*-pre-c-Open sets

Definition 3.1: A s. subset (L, Γ) of S. B.T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ is named s.(1,2)*-pre-c-o. if, for any $\widetilde{m} \tilde{\in} (L, \Gamma)$ where (L, Γ) is s.(1,2)*-pre-o. over H there is a s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. (S,Γ) , and be $\tilde{m} \tilde{\in} (S,\Gamma) \tilde{\subseteq} (L,\Gamma)$. The complement of (L,Γ) is named s. (1,2)*-pre-c-clo.).

The collection of all s. (1,2)*-pre-c-o. set in $\tilde{H}$ is indicate s.(1,2)*-pre-c-o(H) and collection of all S. (1,2)*-pre-c-clo. set in $\tilde{H}$ is indicate s.(1,2)*-pre-c-clo(H).

Remark 3.2: Each s. (1,2)*-pre-c-o. set over H is s. (1,2)*-pre-o. set, but the reverse is not correct generally, seen by the next example:

Example 3.3: Let $H = \{j_1, j_2, j_3\}$ and $\Gamma = \{r_1, r_2\}$, and let $\tilde{\tau}_1 = \{\tilde{H}, \tilde{\emptyset}, (V_1,\Gamma), (V_2,\Gamma)\}$ and $\tilde{\tau}_2 = \{\tilde{H}, \tilde{\emptyset}, (V_3,\Gamma), (V_4,\Gamma)\}$ be s. topologies over H, where

$(V_1,\Gamma) = \{(r_1, \{\tilde{H}\}), (r_2, \{j_2, j_3\})\}$.

$(V_2,\Gamma) = \{(r_1, \{\tilde{\emptyset}\}), (r_2, \{j_1\})\}$.

$(V_3,\Gamma) = \{(r_1, \{j_2\}), (r_2, \{j_1, j_3\})\}$.

$(V_4,\Gamma) = \{(r_1, \{j_2\}), (r_2, \{j_3\})\}$. The soft sets in $\{\tilde{H}, \tilde{\emptyset}, (V_1,\Gamma), (V_2,\Gamma), (V_3,\Gamma), (V_4,\Gamma)\}$ are s. $\tilde{\tau}_1\tilde{\tau}_2$-o. in $\tilde{H}$. Thus $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ is a S. B.T. S.. The s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set are $\tilde{H}, \tilde{\emptyset}, (V_1,\Gamma)^c, (V_2,\Gamma)^c, (V_3,\Gamma)^c, (V_4,\Gamma)^c$ where $(V_1,\Gamma)^c = \{(r_1, \{\tilde{\emptyset}\}), (r_2, \{j_1\})\} = (V_2,\Gamma)$.

$(V_2,\Gamma)^c = \{(r_1, \{\tilde{H}\}), (r_2, \{j_2, j_3\})\} = (V_1,\Gamma)$.
$(V_3,\Gamma)^c = \{(r_1, \{j_1, j_3\}), (r_2, \{j_2\})\}$.

$(V_4,\Gamma)^c = \{(r_1, \{j_1, j_3\}), (r_2, \{j_1, j_2\})\}$.

The combination of s. (1,2)*-pre-o. set $(H) = \{\tilde{H}, \tilde{\emptyset}, (V_1,\Gamma), (V_2,\Gamma), (V_3,\Gamma), (V_4,\Gamma)\}$. And The combination of S. (1,2)*-pre-c-O. set $(H) = \{\tilde{H}, \tilde{\emptyset}, (V_1,\Gamma), (V_2,\Gamma), (V_3,\Gamma)\}$. Then (V_4,Γ) is S. (1,2)*-pre-O. set over H, but (V_4,Γ) is not s. (1,2)*-pre-c-o. Since $(r_1, \{j_2\}) \tilde{\in} (V_4,\Gamma) \tilde{\subseteq}$ s. (1,2)*-pre-o. set. But dosen't s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set contains $(r_1, \{j_2\})$. Therefor (V_4,Γ) is not S. (1,2)*-pre-c-o. set.

Note 3.4: by proposition (3.2) (i) [9] and remark (3.2), we have the following diagram:

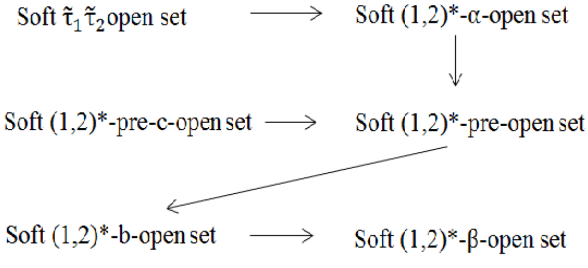

Figure 1: It explains the relationship between open sets

Theorem 3.5: A s. subset (V,Γ) of S. B.T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ is s. (1,2)*-pre-c-o. iff (V,Γ) is s. (1,2)*-pre-o. and it is a union of s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. sets.

Proof: Let (V,Γ) be a s. (1,2)*-pre-c-o. set. Then (V,Γ) is s. (1,2)*-pre-o. set and for every $\tilde{p} \tilde{\in} (V,\Gamma)$ yonder a s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set (L,Γ) , and $\tilde{p} \tilde{\in} (L,\Gamma) \tilde{\subseteq} (V,\Gamma)$. Then we get $\tilde{\cup} \{\tilde{p}\}_{\tilde{p}\tilde{\in}(V,\Gamma)} \tilde{\subseteq} (V,\Gamma) = (L,\Gamma) \tilde{\subseteq} (V,\Gamma)$. so $(V,\Gamma) = \tilde{\cup} (L,\Gamma)$, where (V,Γ) is a s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set for every $\tilde{p} \tilde{\in} (V,\Gamma)$.

$(\Leftarrow)$ forthright by definition s. (1,2)*-pre-c-o..

Corollary 3.6: For a S. B.T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$, if (S,Γ) is s. (1,2)*-pre-o. set over H, then (S,Γ) is s. (1,2)*-pre-c-o. set if (S,Γ) is s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set.

Proof: It is obvious.

Proposition 3.7: An $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$ S. B.T. S. and (L,Γ) is a s. subset on H then (L,Γ) is s. (1,2)*-pre-c-clo. iff (L,Γ) is S. (1,2)*-pre-clo. and be intersection of s. $\tilde{\tau}_1\tilde{\tau}_2$-o. sets.

Proof: It is obvious.
Propositon 3.8: An unplanned union of s.(1,2)*-pre-c-o. set is s.(1,2)*-pre-c-o. set.
Proof: Suppose that $\{(V_\varepsilon,\Gamma)\}_{\varepsilon\in\Omega}$ is a famihy of s. (1,2)*-pre-c-o. sets in $\tilde{U}$. Then $(V,\Gamma)_\varepsilon$ is s. (1,2)*-pre-o. set for every $\varepsilon \in \Omega$. So $\cup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma)$ is s. (1,2)*-pre-o. set. Let $\tilde{x} \tilde{\in} \cup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma) \Longrightarrow \tilde{x} \tilde{\in} (V_{\varepsilon_0},\Gamma)$ for some $\varepsilon \in \Omega$. Since $(V_{\varepsilon_0},\Gamma)$ is a s. (1,2)*-pre-o. set over $\tilde{H}$, then yonder a s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set (S,Γ) over $\tilde{H}$ s. t. $\tilde{x} \tilde{\in} (S,\Gamma)$ and $\tilde{x} \tilde{\in} (S,\Gamma) \tilde{\subseteq} (V_{\varepsilon_0},\Gamma) \tilde{\subseteq} \cup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma)$. So $\tilde{x} \tilde{\in} (S,\Gamma) \tilde{\subseteq} \cup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma)$. Therefor , $\cup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma)$ is s. (1,2)*-pre-c-o. set.
Definition 3.9: Let (M,Γ) be a s. set of a S. B. T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$, then named the s. (1,2)*-pre-c-interior of (M,Γ) (s. (1,2)*-pre-c-int(M,Γ)), if it union to every s. (1,2)*-pre-c-o. sets in $\tilde{H}$ there are include in (M,Γ).

Proposition 3.10: If (M,Γ) be a s. set over H, then s. (1,2)*-pre-c-int$(M,\Gamma) \tilde{\subseteq}$ s. (1,2)*-pre-int(M,Γ).
Proof: Since Every s. (1,2)*-pre-c-o. is s. (1,2)*-pre-O., so the proof holds.

Definition 3.11: Let (M,Γ) be a s. set of a S. B.T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$, then named the s. (1,2)*-pre-c-closure of (M,Γ), denoted by s. (1,2)*-pre-c-cl(M,Γ), if it the intersection of every s. (1,2)*-pre-c-clo. set in $\tilde{H}$ there are include (M,Γ).

In the following theorem we provide some properties to s. (1,2)*-pre-c-interior of s. set.

Theorem 3.12: A S. B.T. S. $(H, \tilde{\tau}_1, \tilde{\tau}_2, \Gamma)$, and (V,Γ),(D,Γ), $(W,\Gamma) \tilde{\subseteq} \tilde{H}$. Then:
(i) s. (1,2)*-pre-c-int$(V,\Gamma) \tilde{\subseteq} (V,\Gamma)$.

(ii) If $\{(V_\varepsilon,\Gamma)\}_{\varepsilon\in\Omega}$ is a combination of s. (1,2)*-pre-c-o. sets in $\widetilde{H}$, then also is $\bigcup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma)$.

(iii) s. (1,2)*-pre-c-int(V,Γ) is the alrgest s. (1,2)*-pre-c-o. set in $\widetilde{H}$ there are include in (V,Γ).

(iv) (D,Γ) is s. (1,2)*-pre-c-o. iff s. (1,2)*-pre-c-int$(D,\Gamma) = (D,\Gamma)$.

(v) (1,2)*-pre-c-int((1,2)*-pre-c-int(V,Γ)) =(1,2)*-pre-c-int(V,Γ).

(vi) $\widetilde{H} -$ ((1,2)*-pre-c-int(V,Γ)) =(1,2)*-pre-c-cl$(\widetilde{H} - (V,\Gamma))$.

(vii) $\tilde{n} \widetilde{\in}$ (1,2)*-pre-c-int(V,Γ) iff yonder a s. (1,2)*-pre-c-o. set (L,Γ) in $\widetilde{H}$ s.t. that $\tilde{n} \widetilde{\in} (L,\Gamma) \widetilde{\subseteq} (V,\Gamma)$.

(viii) So $(V,\Gamma) \widetilde{\subseteq} (W,\Gamma)$, then (1,2)*-pre-c-int$(V,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-int(W,Γ).

(ix) (1,2)*-pre-c-int $((V,\Gamma) \widetilde{\bigcap} (W,\Gamma)) \widetilde{\subseteq}$ (1,2)*-pre-c-int $(V,\Gamma) \widetilde{\bigcap}$ (1,2)*-pre-c-int(W,Γ).

(x) $\bigcup_{\varepsilon\in\Omega}$(1,2)*-pre-c-int$(V_\varepsilon,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-int$(\bigcup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma))$.

Proof:

(i) s. (1,2)*-pre-c-int(V,Γ) =$\widetilde{\bigcup}$ $\{(P_\varepsilon,\Gamma):/(P_\varepsilon,\Gamma)$ is s. (1,2)*-pre-c-o. in $\widetilde{H}$ and $(P_\varepsilon,\Gamma) \widetilde{\subseteq} (V,\Gamma)\}$. Since $(P_\varepsilon,\Gamma) \widetilde{\subseteq} (V,\Gamma) \quad \forall\, \varepsilon \in \Omega \quad \Rightarrow \bigcup_{\varepsilon\in\Omega}(P_\varepsilon,\Gamma) \widetilde{\subseteq} (V,\Gamma)$. Then s. (1,2)*-pre-c-int$(V,\Gamma) \widetilde{\subseteq} (V,\Gamma)$.

(ii) axiomatic by theorem (3.8).

(iii) axiomatic by definiton.

(iv) $(\Longrightarrow)$ $\because (D,\Gamma)$ is a s. (1,2)*-pre-c-o. set in $\widetilde{H}$. We prove that (1,2)*-pre-cint$(D,\Gamma) = (D,\Gamma)$. from (i), we've (1,2)*-pre-cint$(D,\Gamma) \widetilde{\subseteq} (D,\Gamma)$, to prove that $(D,\Gamma) \widetilde{\subseteq}$ (1,2)*-c-int(D,Γ). Since (D,Γ) is s. (1,2)*-pre-c-o. and $(D,\Gamma) \widetilde{\subseteq} (D,\Gamma)$, then by (iii), we get $(D,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-int(D,Γ). Thus (1,2)*-pre-c-int$(D,\Gamma) = (D,\Gamma)$.
$(\Longleftarrow)$ Suppose that (1,2)*-pre-c-int$(D,\Gamma) = (D,\Gamma)$. Since (1,2)*-pre-c-int(D,Γ) is a s. (1,2)*-pre-c-o. set in $\widetilde{H}$, then also (D,Γ).

(v) so (1,2)*-pre-cint(V,Γ) is a s. (1,2)*-pre-c-o. set, then by (iv), we obtain (1,2)*-pre-cint((1,2)*-pre-cint(V,Γ))=(1,2)*-pre-cint(V,Γ)

(vi) Since (1,2)*-pre-cint$(V,\Gamma) = \widetilde{\bigcup}\{(L,\Gamma)$: (L,Γ) is s. (1,2)*-pre-c-o. in $\widetilde{H}$ and $(L,\Gamma) \widetilde{\subseteq} (V,\Gamma)\}$. $\Longrightarrow$ $\widetilde{H} -$ (1,2)*-pre-cint$(V,\Gamma) = \widetilde{H} - \widetilde{\bigcup}\{(L,\Gamma)$: (L,Γ) is s. (1,2)*-pre-c-o. in $\widetilde{H}$ and $(L,\Gamma) \widetilde{\subseteq} (V,\Gamma)\} = \widetilde{\bigcap}\{\widetilde{H} - (L,\Gamma)$: $\widetilde{H} - (L,\Gamma)$ is s. (1,2)*-pre-c-clo. in $\widetilde{H}$ and $\widetilde{H} - (V,\Gamma) \widetilde{\subseteq} \widetilde{H} - (L,\Gamma)\} =$ (1,2)*-pre-c-cl$(\widetilde{H} - (V,\Gamma))$.

(vii) so $\tilde{n} \widetilde{\in}$ (1,2)*-pre-cint$(V,\Gamma) = \widetilde{\bigcup}\{(L,\Gamma)$: (L,Γ) is s. (1,2)*-pre-c-o. in $\widetilde{H}$ and $(L,\Gamma) \widetilde{\subseteq} (V,\Gamma)\}$, then found a s. (1,2)*-pre-c-o. set (L,Γ) in $\widetilde{H}$ such that $\tilde{n} \widetilde{\in} (L,\Gamma) \widetilde{\subseteq} (V,\Gamma)$.
Conv., so there is a s. (1,2)*-pre-c-o. set (L,Γ) in $\widetilde{H}$ such that $\tilde{n} \widetilde{\in} (L,\Gamma) \widetilde{\subseteq} (V,\Gamma)$. But (1,2)*-pre-c-int$(V,\Gamma)$ is the greater s. (1,2)*-pre-c-o. set which include in (V,Γ),.thus $(L,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-int(V,Γ). then $\tilde{n} \widetilde{\in}$ (1,2)*-pre-c-int(V,Γ).

(viii) Impose $\tilde{n} \widetilde{\in}$ (1,2)*-pre-cint(V,Γ), from (vii) we gat a s. (1,2)*-pre-c-o. set (P,Γ) in $\widetilde{H}$ s.t. $\tilde{n} \widetilde{\in} (P,\Gamma) \widetilde{\subseteq} (V,\Gamma)$. Since $(V,\Gamma) \widetilde{\subseteq} (W,\Gamma)$, then there is a s. (1,2)*-pre-c-o. set (P,Γ) in $\widetilde{H}$ such that $\tilde{n} \widetilde{\in} (V,\Gamma) \widetilde{\subseteq} (W,\Gamma)$. Thus $\tilde{n} \widetilde{\in}$ (1,2)*-pre-c-int(W,Γ).

(ix) so $(V,\Gamma) \widetilde{\bigcap} (W,\Gamma) \widetilde{\subseteq} (V,\Gamma)$ and $(V,\Gamma) \widetilde{\bigcap} (W,\Gamma) \widetilde{\subseteq} (W,\Gamma)$, from (viii), we've (1,2)*-pre-c-int$((V,\Gamma) \widetilde{\bigcap} (W,\Gamma)) \widetilde{\subseteq}$ (1,2)*-pre-c-int(V,Γ) and (1,2)*-pre-c-int$((V,\Gamma) \widetilde{\bigcap} (W,\Gamma)) \widetilde{\subseteq}$ (1,2)*-pre-c-int(W,Γ). Thus
(1,2)*-pre-c-int$((V,\Gamma)\widetilde{\bigcap}(W,\Gamma)) \widetilde{\subseteq}$ (1,2)*-pre-c-int$(V,\Gamma) \widetilde{\bigcap}$ (1,2)*-pre-c-int(W,Γ).

(x) Since $(V_\varepsilon,\Gamma) \widetilde{\subseteq} \bigcup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma)$, $\forall\, \varepsilon \in \Omega$. Then by (viii) we get that (1,2)*-pre-c-int$(V_\varepsilon,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-int$(\bigcup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma))$, $\forall\, \varepsilon \in \Omega$.
Thus $\bigcup_{\varepsilon\in\Omega}$(1,2)*-pre-c-int$(V_\varepsilon,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-int$(\bigcup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma))$.

Theorem 3.13: A S. B.T. S. $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$, and (V,Γ), $(D,\Gamma) \widetilde{\subseteq} \widetilde{H}$. Then:

(i) $(V,\Gamma) \widetilde{\subseteq}$ s. (1,2)*-pre-c-cl(V,Γ).

(ii) If $\{(V_\varepsilon,\Gamma)\}_{\varepsilon\in\Omega}$ is a collection of s. (1,2)*-pre-c-clo. sets in $\widetilde{H}$, then so is$\bigcap_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma)$.

(iii) s. (1,2)*-pre-c-cl(V,Γ) is the less s. (1,2)*-pre-c-clo. set in $\widetilde{H}$ there are include (V,Γ).

(iv) (V,Γ) is s. (1,2)*-pre-c-clo. iff s. (1,2)*-pre-c-cl$(V,\Gamma) = (V,\Gamma)$.

(v) (1,2)*-pre-c-cl((1,2)*-pre-c-cl(V,Γ)) =(1,2)*-pre-c-cl(V,Γ).

(vi) $\widetilde{H} -$ (1,2)*-pre-c-cl(V,Γ) =(1,2)*-pre-c-int$(\widetilde{H} - (V,\Gamma))$.

(vii) $\tilde{n} \widetilde{\in}$ (1,2)*-pre-c-cl(V,Γ) iff for every s. (1,2)*-pre-c-o. set (W,Γ) in $\widetilde{H}$ include $\tilde{n}$, such that $(W,\Gamma)\widetilde{\bigcap}(V,\Gamma) \neq \widetilde{\emptyset}$

(viii) If $(V,\Gamma) \widetilde{\subseteq} (D,\Gamma)$, then (1,2)*-pre-c-cl$(V,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-cl(D,Γ).

(ix) (1,2)*-pre-c-cl$((V,\Gamma)\widetilde{\bigcap}(D,\Gamma)) \widetilde{\subseteq}$ (1,2)*-pre-c-cl $(V,\Gamma) \widetilde{\bigcap}$ (1,2)*-pre-c-cl(D,Γ).

(x) $\bigcup_{\varepsilon\in\Omega}$(1,2)*-pre-c-cl$(V_\varepsilon,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-cl$(\bigcup_{\varepsilon\in\Omega}(V_\varepsilon,\Gamma))$.

Proof: (vii) $(\Longrightarrow)$ let it be $\tilde{n} \widetilde{\in}$ (1,2)*-pre-c-cl(V,Γ) and $(W,\Gamma) \widetilde{\bigcap} (V,\Gamma) = \widetilde{\emptyset}$ for some S. (1,2)*-pre-c-O. set (W,Γ) in $\widetilde{H}$ include $\tilde{n}$, so $(V,\Gamma) \widetilde{\subseteq} \widetilde{H} - (W,\Gamma)$.

$\Rightarrow$ $\widetilde{H} - (W,\Gamma)$ is S. (1,2)*-pre-c-clo., from(iv), we've (1,2)*-pre-c-cl$(V,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-cl$(\widetilde{H}$-$(W,\Gamma)) =$

$\widetilde{H} - (W,\Gamma)$. so $\tilde{n} \widetilde{\notin} \widetilde{H} - (W,\Gamma)$ then $\tilde{n} \widetilde{\notin}$ (1,2)*-pre-c-cl(V,Γ). This contr. with the assumption, therefore $(W,\Gamma) \widetilde{\cap} (V,\Gamma) \neq \widetilde{\emptyset}$. for every s. (1,2)*-pre-c-o. set (W,Γ) in $\widetilde{H}$ include $\tilde{n}$.

Conv.Lets prove $\tilde{n} \widetilde{\in}$ (1,2)*-pre-c-cl(V,Γ). if not then $\tilde{n} \widetilde{\notin}$ (1,2)*-pre-c-cl$(V,\Gamma) \Longrightarrow \tilde{n} \widetilde{\in} \widetilde{H}$ −(1,2)*-pre-c-cl(V,Γ) and $\widetilde{H}$ −(1,2)*-pre-c-cl(V,Γ) is a s. (1,2)*-pre-c-o. set. Since $(V,\Gamma) \widetilde{\subseteq}$ (1,2)*-pre-c-cl$(V,\Gamma) \Longrightarrow (V,\Gamma) \widetilde{\cap} (\widetilde{H}$ −(1,2)*-pre-c-cl$(V,\Gamma)) = \widetilde{\emptyset}$. There is discrepancy. then $\tilde{n} \widetilde{\in}$ (1,2)*-pre-c-cl(V,Γ). from def. (3.11) , other cases can we prove.

Remark 3.14:. The intersection. of two s. (1,2)*-pre-c-o. set over H is not. necessarily s. (1,2)*-pre-c-o. set, by the following example:

Example 3.15: In S. B.T. S. $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$, as in example (3.3) (V_1,Γ) and (V_3,Γ) is s. (1,2)*-pre-c-o. sets, but $(V_1,\Gamma) \widetilde{\cap} (V_3,\Gamma) = (V_4,\Gamma)$ is not. s. (1,2)*-pre-c-o. set.

Proposition 3.16:. If $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ is a S. B.T. S. with property (I), then the collection of all s. (1,2)*-pre-c-o. sets in $\widetilde{H}$ froms a s. topology on $\widetilde{H}$.

Proof: (i) It is clear that $\widetilde{H}$ and $\widetilde{\emptyset}$ are s. (1,2)*-pre-c-o. sets in $\widetilde{H}$.

(ii) Let (V_1,Γ) and (V_2,Γ) be s. (1,2)*-pre-c-o. sets in $\widetilde{H}$. To prove that $(V_1,\Gamma) \widetilde{\cap} (V_2,\Gamma)$ is s. (1,2)*-pre-c-o. Let $\tilde{p} \widetilde{\in} (V_1,\Gamma) \widetilde{\cap} (V_2,\Gamma) \Longrightarrow \tilde{p} \widetilde{\in} (V_1,\Gamma)$ and $\tilde{p} \widetilde{\in} (V_2,\Gamma)$. Since (V_1,Γ) is s. (1,2)*-pre-c-o.$\Longrightarrow \exists$ (W_1,Γ) is a s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set in $\widetilde{H}$ s.t. $\tilde{p} \widetilde{\in} (W_1,\Gamma)$ and $\tilde{p} \widetilde{\in} (W_1,\Gamma) \widetilde{\subseteq} (V_1,\Gamma)$. Since (V_2,Γ) is s. (1,2)*-pre-c-o.$\Longrightarrow \exists$ (W_2,Γ) is a s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set in $\widetilde{H}$ s.t. $\tilde{p} \widetilde{\in} (W_2,\Gamma)$ and $\tilde{p} \widetilde{\in} (W_2,\Gamma) \widetilde{\subseteq} (V_2,\Gamma)$. Since $\tilde{p} \widetilde{\in} (W_1,\Gamma)$ and $\tilde{p} \widetilde{\in} (W_2,\Gamma) \Longrightarrow \tilde{p} \widetilde{\in} (W_1,\Gamma) \widetilde{\cap} (W_2,\Gamma)$ and $(W_1,\Gamma) \widetilde{\cap} (W_2,\Gamma)$ is s. $\tilde{\tau}_1\tilde{\tau}_2$-clo.. Since $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ with property (I). Then $\tilde{p} \widetilde{\in} (W_1,\Gamma) \widetilde{\cap} (W_2,\Gamma) \widetilde{\subseteq} (V_1,\Gamma) \widetilde{\cap} (V_2,\Gamma)$. Hence $(V_1,\Gamma) \widetilde{\cap} (V_2,\Gamma)$ is s. (1,2)*-pre-c-o.

(iii) Follow from proposition (3.8).

Therefore from (i),(ii) and (iii) we get that the collection of s.(1,2)*-pre-c-o. sets in $\widetilde{H}$ with property (I) froms a S. T. on $\widetilde{H}$.

Theorem 3.17: If $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ is a S.(1,2)*-$\widetilde{T}_1$-S., then (L,Γ) s.(1,2)*-pre-o. set iff (L,Γ) s.(1,2)*-pre-c-o. set.

Proof: ∵ $(H,\tilde{\tau}_{1.},\tilde{\tau}_{2.},\Gamma)$ is a S.(1,2)*-$\widetilde{T}_1$-S., and (L,Γ) s. (1,2)*-pre-o. set. If $(L,\Gamma) = \widetilde{\emptyset}$ then (L,Γ) s. (1,2)*-pre-c-o. set in $\widetilde{H}$. If $(L,\Gamma) \neq \widetilde{\emptyset}$, let $\tilde{n} \widetilde{\in} (L,\Gamma)$. since$(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ be S. (1,2)*-$\widetilde{T}_1$-S., then by th. (1.12) each s. point is s. $\tilde{\tau}_1\tilde{\tau}_2$-clo. set, and $\tilde{n} \widetilde{\in} \{\tilde{n}\} \widetilde{\subseteq} (L,\Gamma)$. so there (L,Γ) s. (1,2)*-pre-c-o. set, thus the s.(1,2)*-pre-o.(H) $\widetilde{\subseteq}$ s.(1,2)*-pre-c-o.(H). But s.(1,2)*-pre-c-o.(H) $\widetilde{\subseteq}$ s.(1,2)*-pre-o.(H). Hence s.(1,2)*-pre-o.(H) = s.(1,2)*-pre-c-o.(H).

Proposition 3.18: If $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ is a s. locally indiscrete, then the s. (1,2)*-semi-o. set in $\widetilde{H}$ is s. (1,2)*-pre-c-o. set in $\widetilde{H}$.

Proof: Let (W,Γ) be any s. subset of $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ and (W,Γ) is s. (1,2)*-semi-o. set in $\widetilde{H}$, If $(W,\Gamma) = \widetilde{\emptyset}$ then (W,Γ) s. (1,2)*-pre-c-o. set in $\widetilde{H}$. $(W,\Gamma) \neq \widetilde{\emptyset}$, then $(W,\Gamma) \widetilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(W,\Gamma))$. Since $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ is a s. locally indiscrete, then $\tilde{\tau}_1\tilde{\tau}_2 int(W,\Gamma)$ is s. $\tilde{\tau}_1\tilde{\tau}_2 -$ clo., so $\tilde{\tau}_1\tilde{\tau}_2 int(W,\Gamma) \widetilde{\subseteq} (W,\Gamma)$. This implies that for each and $\tilde{n} \widetilde{\in} (W,\Gamma)$, then $\tilde{n} \widetilde{\in} \tilde{\tau}_1\tilde{\tau}_2 int(W,\Gamma) \widetilde{\subseteq} (W,\Gamma)$. Therefore (W,Γ) s. (1,2)*-pre-c-o. set in $\widetilde{H}$.Hence the s. (1,2)*-semi-o. set is s. (1,2)*-pre-c-o. set in$\widetilde{H}$.

Theorem 3.19: If $\{(M_\varepsilon,\Gamma)\}_{\varepsilon\in\Omega}$ be a collection of s. (1,2)*-pre-c-clo. set of S. B.T. S. $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$. Then $\bigcap_{\varepsilon\in\Omega}(M_\varepsilon,\Gamma)$ is s. (1,2)*-pre-clo. set .

Proof: it's axiomatic by proposition (3.8).

Definition 3.20:. Let it $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ be S. B. T. S. and $\tilde{n} \widetilde{\in} \widetilde{H}$. a s. set (L,Γ) is said to be s.(1,2)*-pre-c-neighborhood of $\tilde{n}$, if there is found a S. (1,2)*-pre-o. set (O,Γ) over H s.t. $\tilde{n} \widetilde{\in} (O,\Gamma) \widetilde{\subseteq} (L,\Gamma)$.

From example (3.3) $(N,\Gamma) = \{(r_1,\{j_2,j_3\}),(r_2,\{j_1,j_3\})\}$ is soft (1,2)*-pre-c-nbh. of a soft point $\tilde{n} = (r_1,\{j_2\}) \widetilde{\in} \widetilde{H}$, but $(M,\Gamma) = \{(r_1,\{j_2,j_3\}),(r_2,\{j_2\})\}$ is not soft (1,2)*-pre-c-nbh. of a soft point $\tilde{n}$.

Proposition 3.21: Let $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ be S. B.T. S., and a s. set (D,Γ) is s. (1,2)*-pre-c-o. iff it is a s. (1,2)*-pre-c-nbh. of every of its s. points.

Proof: impose $(D,\Gamma) \widetilde{\subseteq} \widetilde{H}$ be a s. (1,2)*-pre-c-o. set, ⟹for each $\tilde{q} \widetilde{\in} (D,\Gamma)$, $\tilde{q} \widetilde{\in} (D,\Gamma) \widetilde{\subseteq} (D,\Gamma)$ and (D,Γ) is s. (1,2)*-pre-c-o. set, this shows that (D,Γ) is a s. (1,2)*-pre-c- nbh. of every of its s. points.

Conv.: so that (D,Γ) is a s. (1,2)*-pre-c-nbh. of every of its s. points. ⟹ for every $\tilde{q} \widetilde{\in} (D,\Gamma)$, thing exists $(K,\Gamma)_{\tilde{q}}$ is s. (1,2)*-pre-c-o. set in $\widetilde{H}$ s.t. $(K,\Gamma)_{\tilde{q}} \widetilde{\subseteq} (D,\Gamma)$. ⟹ $(D,\Gamma) = \widetilde{\cup} \{(K,\Gamma)_{\tilde{q}} : \tilde{q} \widetilde{\in} (D,\Gamma)\}$. that any $(K,\Gamma)_{\tilde{q}}$ is s. (1,2)*-pre-c-o. set. then (D,Γ) is s. (1,2)*-pre-c-o. set.

Proposition 3.22: Every s. (1,2)*-pre-c- nbh. of a s. point is s. (1,2)*-pre- nbh..

Proof: It's axiomatic from remark (3.2),.

Conclusions

During our study of the treatise. We have got many important results and facts in S. B.T. S., as well as important results and facts in the subject of s. (1,2)*-pre-c-o. set in S. B.T. S.. Also we define some important concepts and we obtained important results. Then we will mention some of these resulte they reached in this treatise :

- A soft subset (M,Γ) of S. B.T. S. $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ is s. (1,2)*-pre-c-o. iff (M,Γ) is s. (1,2)*-pre-o. and it is a union of S. $\tilde{\tau}_1\tilde{\tau}_2$-C. sets.
- Each s. (1,2)*-pre-c-o. set over H is s. (1,2)*-pre-o. set, but the converse is not correct generally.
- If $(H,\tilde{\tau}_1,\tilde{\tau}_2,\Gamma)$ is a S. B.T. S. with property (I), then the collection of all s. (1,2)*-pre-c-o. sets in $\widetilde{H}$ froms a soft topology on $\widetilde{H}$.
- Every s. (1,2)*-pre-c- nbh. of a S. point is s. (1,2)*-pre-nbh.

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