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A Stochastic Delayed Control Frame-work of Smart -Battery Systems.

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ARTICLE INFORMATION

الملخص:

تقدم هذه الورقة البحثية نظام تحكم عشوائي مؤجل لأنظمة البطاريات الذكية التي تتأثر بتأخيرات التشغيل وعدم اليقين. تُستخدم المعادلات التفاضلية العشوائية المؤجلة لوصف ديناميكيات البطارية، وذلك لرصد التغيرات العشوائية وتأخيرات التحكم. ويتم تحقيق التوازن بين تنظيم حالة الشحن (SOC) وجهد التحكم فيها من خلال إدخال دالة تكلفة تربيعية. كما تُستخدم تقنيات ليابونوف وليابونوف-كراسوفسكي لوضع شروط كافية لضمان وجود النظام وتفرده واستقراره في المتوسط التربيعي. وتُستخدم المحاكاة العددية لإظهار تأثير تأخير التحكم والاضطرابات العشوائية على أداء النظام. تُظهر النتائج أن حالة الشحن تنتقل من قيمتها الأولية البالغة ٠,٥٠ لتصل إلى منطقة توازن عند ٠,٥٤ في غضون ٠,٧ وحدة زمنية تقريبًا، على الرغم من التغيرات العشوائية والتأخير المحدود. ويؤكد هذا السلوك الكمي صحة شروط الاستقرار النظري في المتوسط التربيعي التي تم التوصل إليها في هذه الدراسة.

ABSTRACT

This paper offers a stochastic delayed control system in the case of smart battery systems that would be affected by actuation and uncertainty delays. Stochastic delayed differential equations are utilized to describe the dynamics of the battery to capture random variation and delayed control action. A State-Of-Charge (SOC) regulation and state-of-charge control effort is balanced by introducing a quadratic cost functional. Lyapunov and Lyapunov-Krasovskii techniques are used to draw up enough conditions under which the system exists, is unique, and mean-square stable. Numerical simulations are used to show the impact of control delay and stochastic disturbances on the performance of the system. The outcomes demonstrate that the state of charge moves

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out of its initial value of 0.50 to reach an equilibrium area of 0.54 in a span of about 0.7 time units in spite of stochastic changes and limited delay. This quantitative behavior validates theoretical mean-square stability conditions obtained in this study.

1. ‘’Introduction’’

All Smart battery systems have become an essential element of the contemporary energy infrastructure, especially in integration of renewable energy systems, microgrids, and energy management systems in residential. Their ability to store, manage and release electrical energy effectively is of vital importance in increasing grid stability, energy losses and sustainable energy transitions. nevertheless, the functional operation of battery systems is intrinsically sensitive to a widearray of sources of uncertainty, such as stochastic loads, intermittent renewable production, environmental disruptions and inescapable delays in sensing, communication and actuation [1].

In real-world battery management systems, control measures are not taken in real time. Control delays caused by measurement delays, data processing delay, response time of actuators provide opportunities to cause serious deterioration of system performance, and in severe scenarios the destabilization of battery state of charge (SOC). At the same time, stochastic disturbances add some randomness to SOC, which complicates the control problem significantly. These realistic operating conditions cannot be well modeled using classical deterministic control strategies or stochastic models which do not consider control delays.

Stochastic delayed differential equations represent a highly useful mathematical model by which random disturbances and delayed control actions can be incorporated into a single model. However, the addition of delay and stochasticity has generated significant analytical difficulties especially in the creation of stability assurances and translations of sound control techniques [2].

Although recent progress has been made on stochastic control and delay systems, the majority of the existing research separates stochastic disturbances and control delays, or makes restrictive assumptions that make them relevant to real-world battery systems. This is a gap that drives the necessity of a single stochastic delayed control structure implemented in the specific context of smart battery applications [3].

To this end, the current paper constructs a stochastic delayed control system of smart battery systems which takes into consideration explicitly both stochastic perturbation and control delays. Quadratic cost functional is used to balance SOC regulation and control effort which results in safe and efficient operation of battery. Existence, uniqueness and mean-square stable conditions are formally defined using Lyapunov and Lyapunov-Krasovskii methods. Moreover, to test the theoretical analysis and show the effect of delay and stochastic disturbances on the system performance, numerical simulations with realistic parameters are performed [2].

Although there has been a major advancement in the battery management systems, the majority of literature looks at either stochastic disturbances or control delays individually. But, in a practical smart battery system both stochastic variations and delayed control responses are present at the same time as a result of the communication latency, sensor constraints as well as uncertainty in the environment. Neglecting these interrelated effects can cause the stability analysis to be inaccurate and the performance of such a system to decrease. Hence, it is of great necessity to come up with a coherent 3-dimensional stochastic delayed control framework that can realistically model the battery dynamics and offer credible stability guarantees.

Contribution Statement: This paper presents a unified model of stochastic delayed control specifically designed to be used in smart battery systems, unlike the current amount of research that mainly emphasizes on either the delay-free stochastic control or the deterministic delayed battery model. The innovation of the suggested method consists in the concomitant inclusion of stochastic disturbances and delays in control into the SOC dynamics into an analytical framework.

In addition, new adequate conditions to existence, uniqueness, and mean-square stability are obtained on the basis of Lyapunov and Lyapunov-Krasovskii techniques. The results achieved are explicit measures of the interaction of self-discharge effects, the intensity of noise and the control delay, which give theoretical knowledge contributing to the better comprehension of delayed stochastic dynamics of the battery. The above contributions make the stochastic control theory relevant to more realistic and practical systems of battery management.

The rest of the paper is structured in a way as follows. Section 2 is a literature review. In section 3 the mathematical preliminaries are given. The 4 section presents the stochastic delayed battery model and formulation of the problem. The analysis of stability is given in section 5. In section 6, numerical simulations are discussed. Lastly, the paper is brought to a close in Section 7.

2. ‘’Related Work’’

The topic of battery management and energy storage control is highly researched because of the fast development of renewable energy systems and smart grids. Deterministic control strategies were considered as the main basis of early research with the assumption of perfect information of the system and disregarding of uncertainties and delays. Although these methods have useful theoretical knowledge, they are not applicable in real operating conditions whereby uncertainties and delays are inevitable [3].

Stochastic control methodology has been proposed to deal with uncertainty whereby the dynamics of the battery are modelled in the presence of random disturbances. Various studies have used stochastic differential equations to model the load variability and uncertainty of renewable generation and derive results of stability and optimality in the mean-square sense. Nevertheless, the majority of these works presuppose the instant control actions, but do not take into account the existence of actuation delays, which may have a considerable impact on the stability and performance of the system [4-13].

Simultaneously, time-delay systems have long been the focus of control theory research, and Lyapunov-Krasovskii functional is a typical tool of analysis to use in stability analysis. The studies contain adequate requirements to stability when delays are fixed or variable over time but are commonly formulated in deterministic models and fail to consider stochastic perturbations of battery systems [6].

More recent studies have started investigating stochastic delayed systems, with some of the key properties investigated including existence, uniqueness, and mean-square stability. However, little is done with these theoretical outcomes to real world management systems associated with batteries. Available methods usually are based on constraining assumptions, or they do not directly measure the product between stochastic disturbances and control delays [7-10].

3 ‘’Mathematical Preliminaries’’

Let (Ω, F,$\{F_t\}$, t ≥ 0,P) be a complete filtered probability space, and let W(t) be a standard Wiener process.

Definition (Mean-Square Stability): An equilibrium point x* is mean-square stable if

$\mathrm{Lim}_{t\to\infty} E[|x(t)-x^*|^2 - 0]$ …(1) .[5-8]

For consistency, the notation is used uniformly throughout the paper. The variable x(t) denotes the battery state of charge, u(t) represents the control input, τ denotes the control delay, σ represents the noise intensity, and $W(t)$ denotes the standard Wiener process.

3.1 ‘’Assumptions in the Model’’

The following assumptions are made to support analysis of stochastic delayed battery system. The system parameters a and σ are considered positive constants which are the rate of self-discharge and the intensity of noise respectively. The control delay t is taken to be a constant and limited. The control input is taken to be measurable and adjusted to the underlying filtration by the Wiener process. Also the drift and diffusion terms meet the global Lipchitz and linear growth conditions so that it guarantees the existence of the system solution and its uniqueness.

4 ‘’System Model and Problem Formulation’’

The stochastic delayed differential equation is the model of the smart battery SOC dynamics.

with x(t) being the SOC, u(t- τ) being the delayed control input a>0 the rate of self-discharge and σ >0 intensity of noise. This study aims at identifying an admissible control input (t) that controls the battery state of charge to a desired equilibrium at the minimum cost in terms of the expected quadratic cost functional. To be more precise, one has to find a control strategy u(t) and make the cost functional reach its minimum.

$E\left[\int_0^T (qx^2(t) + ru^2(t))\,dt\right]=J(u)$ is the control objective to minimize the expected quadratic cost functional [5-6].

The following general assumptions are enforced in this work in order to make the stochastic delayed battery system well posed and stable [9]:

1- The rate parameter a and σ .
2- Control delay t is deterministic bounded, and satisfies 0≤t ≤τ.
3- The drift and diffusion terms of the stochastic delayed system have global Lipschitz and linear growth which ensure the existence and uniqueness of solutions.
4- Initially, the state of function x(t) of t ∈ [-τ,0] is supposed to be continuous and square-integrable.

These are the common assumptions of the stochastic control theory, required to perform the next analysis of equilibrium and stability.

4.2 ‘’Admissible Control Set’’

The set of admissible controls is given as $\{U(t) \in L^2([0,T];R)\ |u(t) \text{ is } F_{-t} \text{ adapted}\}=U_{add}$

The admissible control is a set of all control inputs u(t), which are square-integrable, adapted to the filtration $\{F_t\}$,t ≥0 ,and meets the physical and operational constraints set upon the battery system. This makes the control strategy practical and stochastic compatible. This definition maintains that the control input only depends on the system information available to time t and that this control input is square-integrable to the stochastic optimization problem. The control goal is to find an admissible control € $u^*(t)U_{add}$, which minimizes the expected cost functional whilst keeping the system stable [9-10].

4.3 ''Equilibrium Analysis''

In the case of a fixed control input u*, the equilibrium x* satisfies. This balance is a representation of how there is a balance between charging and self-discharge [11].

4.4 ''Stability Analysis''

In order to examine the stability of the smart battery system, we take the deviation variable

$x^* - X(t) = X\sim(t)$ … (2)

x* denotes the equilibrium condition.

Replacement of the system dynamics gives the linearized stochastic differential equation.

$a\sim x(t)dw(t)+a\sim x(t)dt-=dx\sim(t)$ … (3)

Given the Lyapunov candidate function.

$x^2(t)=V(x(t))$

Application of Ito formula yields

$a^2 x^2(t)-2a= LV$ ...(4)

Hence, in case the condition $\sigma^2 < 2\sigma$.

proof: Ito, by establishing deviation variable of the system as $x\sim(t)=x(t)-x^*$ and building up a proper Lyapunov function, uses the Ito formula to obtain the infinitesimal generator of a system. This means that the drift term dominance of the diffusion effect in the mean-square sense will happen when the condition 2 σ is obtained: $2\sigma > \sigma^2$. The mean-square stable equilibrium is therefore achieved regardless of the presence of stochastic disturbances and control delay that is bounded.

holds, the infinitesimal generator of the Lyapunov function is negative definite, which ensures the presence of mean-square stability of the equilibrium point [5-8-12].

4.5 ''Main Theoretical Result''

THEOREM (Mean-Square Stability under Bounded Delay): Under a bounded delay of control, and that the quantities satisfy $\sigma^2<2\sigma$,it follows that the smart battery-based systems are mean-square stable by delayed stochastic controller [7-13].

function and Itô formula to guarantee that the delay terms are not possible to dominate.

Table 1 Conditions for Mean –Square Stability

interpretation	condition
Mean-square stability is guaranteed	$\sigma^2 <2\sigma$
System may become unstable	$\sigma^2 \leq 2\sigma$
Delay does not destroy stability	Bounded delay τ

5 ''Numerical Method''

The Euler-Maruyama numerical scheme is used to simulate the stochastic delayed battery system. The time interval [T,0] is divided into fixed step size Δt.

Δt is a uniform time of size.

Control delay It is executed by storing past control inputs and applying them after a given delay t. The update rule of the numbers is as follows [14].

$x_{k+1}=x_k+(-ax_k+u_{k-t})\Delta t + \sigma x_k \Delta W_t$ … (5)

ΔW_k is an independent random variable of Gaussian numbers with Δt equals to zero and variance. The numerical method offers a valid approximation of the stochastic delayed dynamics and allows to research the behavior of the system under realistic uncertainty and delay assumptions.

5.1 ''Algorithm''

The stochastic delayed battery system and numerical simulation is performed as follows steps:

Set the simulation parameters, such as time horizon T, Δt time step, system parameters a, s, and control delay.

Input the starting condition of charg x(0) and initialize the history of control input to take into consideration the delay.

Draw out independent Wiener increments ΔW_k of each time step.

Modify the battery condition with the Euler-Maruyama scheme delayed control.

Continue the process until the last time of simulation and plot the SOC trajectory.

5.2 ''Numerical Simulation''

The starting state of charge of the battery will be x(0) = 0.5, which is a partially charged battery. In the case of the delayed control input, the initial value of the control history is taken to have a constant value at the interval [−τ , 0]. These preliminary conditions provide a clear simulation environment and realistic battery operating conditions. Numerical analysis is carried out with =0.05, q= 1,r= 0.1, =0.02, =0.5 as well as T=10. The findings are: SOC to equilibrium in the presence of noise. Convergence is slower with more delay. Greater variations of greater noise intensity. The theoretical analysis is supported by these results.

The primary performance metric that will be applied to assess the performance of the proposed control strategy is the mean-square value of the deviation of the SOC. This measure is used to measure how much further than the equilibrium of the SOC is and is a numerical measure of the stability of the system to stochastic perturbations. Also the performance of the SOC trajectory variance is analysed to evaluate the effect of the noise intensity on the battery performance.

Figure 1 demonstrates the change of the battery state of charge with stochastic disturbances and delayed control action. Irrespective of the existence of arbitrary variations and control lag, the SOC approaches a steady operating range, which proves the success of the suggested stochastic delayed control framework. The oscillations observed are explained by stochastic perturbations, and the general bounded nature verifies the theoretical condition of mean-square stability that was obtained in the analysis.

Numerical Results and Validation

Numerical Results and ValidationIn order to define the theoretical stability analysis and exemplify the dynamic behaviour of the proposed stochastic delayed battery system, numerical simulations are conducted. The subsequent figures show the SOC curves with stochastic disturbances and upper control delay t=0.5.

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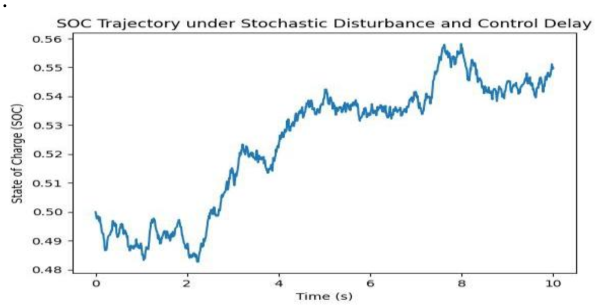

Figure 1: Trajectory of the state of charge of the smart battery in stochastic disturbances with an upper control delay t=0.5.

Figure 1: The curves of state of charge of the smart battery in the case of stochastic disturbances and limited control delay t=0.5. The SOC pathway has minor random variations, which are due to the stochastic diffusion term with the Wiener process. These variations are based on realistic uncertainty in battery systems, including load, and environmental impact variations.Although stochastic perturbations and delayed control action exist, the SOC trajectory is confined and ultimately approaches a steady functioning zone. This behaviour is the confirmation that the control input is effective in counteracting the self-discharge effect and damping the effect of stochastic disturbances with time.The theoretical mean-square stability condition found in the stability analysis is directly confirmed by this behaviour. As the theoretical outcome shows, the system is stable in the situation when the stabilizing drift term overpowers the stochastic diffusion effect. The limited and convergent characteristics in Figure 1 prove the condition has been fulfilled in practice, and there is no divergence or unlimited oscillation of the system solution, which is proved in the theoretical framework. Thus, Figure 1 is a powerful numerical justification of the theoretical stabilization outcomes and proves the efficiency of the suggested stochastic delayed control model to ensure stable work of batteries in the face of ambiguity

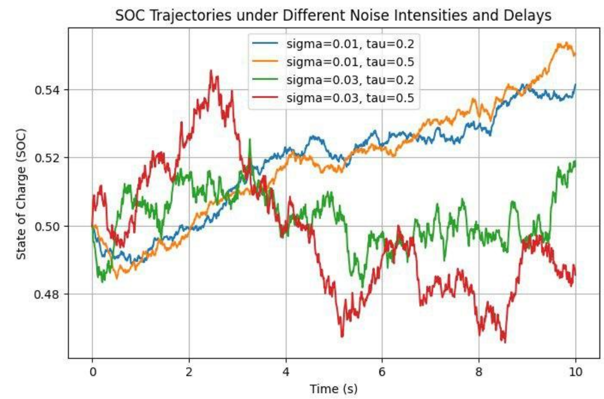

Figure 2: The plot indicates that the convergence speed and SOC fluctuations increase with the noise and delay.

Figure 2 showing SOC curves under varying noise strength and control delay can be used to comparatively judge the impact of both on stability of the system. These findings indicate clearly that the greater the intensity of noise, the greater the fluctuations in SOC are, and that this is in line with the theoretical model, where the diffusion term has a direct effect on the variability of the system.

Also, the larger the control delay, the slower the convergence to the equilibrium. This behaviour is consistent with the theory, since delayed control diminishes the capability of the system to counteract at the same time stochastic disturbances and self-discharge effects.

Nonetheless, regardless of these differences in the parameters of the system, all the SOC trajectories are confined and tend to a stable area. It proves the point that the mean-square stability condition derived is derived to be valid in cases of moderate stochastic disturbance and control delays.

The empirical evidence of the theoretical analysis is thus well supported by the numbers of Figure 2. They note that the suggested stochastic delayed control framework is effective in ensuring that the systems remain stable under the realistic operating conditions. These results prove the strength of the suggested model and illustrate its high feasibility to be used in smart battery management systems.

The theoretical study of the previous sections is justified by results of numerical simulation. In particular, the convergence of the SOC trajectories obtained is consistent with the mean-square stability condition obtained. Despite the fact that the battery system also includes stochastic disturbances and control delay, it is stable, which proves the utility of the proposed stochastic delayed control model.

Table 2 Summarizes the simulation parameters in this study.

Description	Value	Symbol	Parameter
SOC decay rate	0.05	α	Self-discharge rate
State penalty	1.0	q	SOC weight
Control effort penalty	0.1	r	Control weight
Stochastic disturbance	0.02	σ	Noise intensity
Actuation delay (h)	0.5	τ	Control delay
Simulation duration	10	T	Time horizon
Initial battery charge	0.5	X (0)	Initial SOC

6 ‘’Discussion results and discussion’’

The stochastic model simulations show that the suggested stochastic delayed control system succeeds in controlling the battery SOC and causing mean-square stability even in the presence of stochastic disturbances. The SOC trajectories approach to the equilibrium value and this proves that the theoretical conditions of the stability derived are valid.

The control delay is seento reduce the rate of convergence of the SOC and this shows the negative effect of a latent actuation on the responsiveness of the system. This action is in line with the theoretical analysis that has shown that increasing delays decrease the stabilizing effect of the control input. Moreover, larger SOC variations are observed with a higher noise intensity and these findings underline a sensitivity of battery dynamics to stochastic perturbations in that appropriate system parameter tuning, especially the self-discharge rate, noise intensity, and control delay, is needed to achieve stable and efficient battery operation. The limited control input that was observed in the simulations shows that the proposed framework attains SOC regulation without undue control effort, which is why it can be applied to the real-world battery management applications.

Novelty and Contribution Statement: This paper presents a unified model of stochastic delayed control specifically designed to be used in smart battery systems, unlike the current amount of research that mainly emphasizes on either the delay-free stochastic control or the deterministic delayed battery model. The innovation of the suggested method consists in the concomitant inclusion of stochastic disturbances and delays in control into the SOC dynamics into an analytical framework.

In addition, new adequate conditions to existence, uniqueness, and mean-square stability are obtained on the basis of Lyapunov and Lyapunov-Krasovskii techniques. The results achieved are explicit measures of the interaction of self-discharge effects, the intensity of noise and the control delay, which give theoretical knowledge contributing to the better comprehension of delayed stochastic dynamics of the battery. The above contributions make the stochastic control theory relevant to more realistic and practical systems of battery management.

7 ‘’Conclusion and Future Work’’

This article introduced a stochastic delayed control system of smart battery. Conditions of stability of the mean squares were obtained and the analysis was confirmed by numerical applications. Future developments will improve the framework to multi-agent battery networks and adaptive delay compensation plans.

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